Probabilistic Fair Ordering of Events

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ABSTRACT

A growing class of applications depends on fair ordering, where events that occur earlier should be processed before later ones. Providing such guarantees is difficult in practice because clock synchronization is inherently imperfect: events generated at different clients within a short time window may carry timestamps that cannot be reliably ordered. Rather than attempting to eliminate synchronization error, we embrace it and establish a probabilistically fair sequencing process. Tommy is a sequencer that uses a statistical model of per-clock synchronization error to compare noisy timestamps probabilistically. Although this enables ordering of two events, the probabilistic comparator is intransitive, making global ordering non-trivial. We address this challenge by mapping the sequencing problem to a classical ranking problem from social choice theory, which offers principled mechanisms for reasoning with intransitive comparisons. Using this formulation, Tommy produces a partial order of events, achieving significantly better fairness than a Spanner TrueTime-based baseline approach.

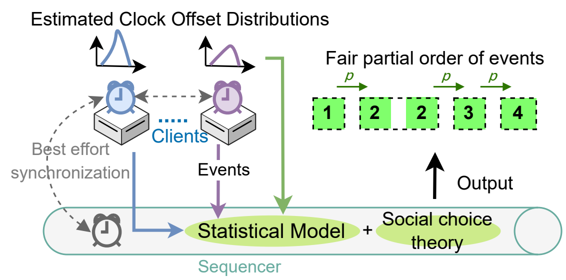

Figure 1: The sequencer, Tommy, uses clock correction distributions & timestamps to achieve a fair ordering of events.

1 INTRODUCTION

Sequencers are fundamental building blocks in distributed systems, providing a mechanism to impose a total order on events that may occur at different locations. They underpin many core protocols, including consensus and concurrency control. In classical consensus protocols such as Paxos [1] and Raft [2], a leader implicitly acts as the sequencer, both determining a total order of operations and coordinating agreement on that order. More recently, network-based sequencers have been proposed to offload sequencing responsibilities from higher level protocols. Systems such as NOPaxos [3], Hydra [4], NeoBFT [5], and SwitchBFT [6] explicitly decouple sequencing from other protocol logic, introducing dedicated sequencer components to improve efficiency.

At a high level, the role of a sequencer is straightforward: assign ranks to incoming messages to establish a total order for processing. In most existing systems, this ranking is independent of when a message was generated. Instead, messages are ordered according to when they are observed by the sequencer, effectively implementing a FIFO policy. For many traditional applications, this behavior is sufficient, since the system only requires *some* ordering, even if that ordering is arbitrary.

A growing class of applications requires a stronger notion of ordering, which we refer to as fair ordering. Unlike FIFO ordering, *fair ordering requires that an event generated earlier (in terms of wall-clock time) be processed before an event generated later.* FIFO ordering approximates this goal only under limited deployment circumstances e.g., specially engineered networks deployed by high-frequency trading exchanges where clients are connected to the server with meticulously equalized data-path lengths. However, generally FIFO ordering is not a proxy for fair ordering of events.

The need for fair ordering is particularly pronounced in financial exchanges, ad exchanges, and other competitive systems [7–13], where ordering decisions directly affect economic outcomes. We refer to such systems as *auction apps*. In these environments, hundreds of clients can generate millions of events within a narrow time window following a sensitive trigger, such as a market moving announcement that is broadcast simultaneously to all participants [8, 9, 14, 15]. In this setting, ensuring that messages generated earlier are processed before those generated later is essential for preserving fairness. The absence of a practical fair ordering primitives is a key reason why many financial exchanges continue to operate in private and specialized data centers incurring high costs, rather than using cheaper public cloud infrastructure [15].

Prior work has recognized the need for fair ordering in auction apps, but existing solutions either rely on strong and often impractical assumptions, such as near perfect clock synchronization [8, 14], or are tightly coupled to the logic of a specific application, limiting their generality [9]. In this paper, we describe a general abstraction for fair ordering, which we term a *fair sequencer*. A fair sequencer ensures that, with high probability, an earlier event is assigned a lower rank than a later event.

Classical Context: Ordering of events is a well-studied problem. Lamport's foundational work [16] introduced the *happened-before* relation ($\rightarrow$). If two events are causally related, meaning one can influence the other, then they can be ordered using this relation. The happened-before relation is transitive, allowing a set of pairwise causally related events to be partially ordered. Events that are concurrent, for which no causal relationship can be established, remain unordered. We generalize the ordering relation to account for such events to achieve fair ordering.

Fundamental Challenge: Achieving ideal fair ordering would require perfect clock synchronization, so that timestamps of the events occurring at different clients can be directly compared without being concerned about the datapath length experienced by the messages carrying information about events. However, perfect clock synchronization is impossible in asynchronous or bounded synchronous networks [17, 18] due to fundamental uncertainty in communication delays. In particular, it is impossible to synchronize the clocks of n processes more tightly than $U(1-1/n)$, where U denotes the uncertainty in link delays [18]. This inherent limitation makes achieving fair ordering a challenge.

Our Approach: We build on the insight that *two noisy timestamps from two clients can be meaningfully compared if the distributions of their clock offsets relative to a reference clock are known.* Using the insight, we introduce a new relation, *likely happened before*, denoted $\xrightarrow{p}$, where p represents the probability that one event occurred before another. Similar to the classical *happened-before* relation, $\xrightarrow{p}$ induces a partial order, but one that also captures order fairness albeit probabilistically. Figure 1 illustrates the architecture where the event timestamps and clock offsets distributions are shared with the sequencer that achieves a fair order of events using a statistical model and tools from social choice theory, as described below.

There are two central challenges in using the $\xrightarrow{p}$ relation for fair sequencing beyond constructing the relation for any two events. First, unlike the *happened-before* relation, $\xrightarrow{p}$ is not necessarily transitive, which complicates ordering more than two events. Second, the sequencing must be performed in an online fashion i.e., a sequencer cannot assume that all events that need to be ordered have already been seen by the sequencer. An event to be seen by the sequencer in the future may need to be ordered before an event seen earlier. This complicates the sequencing in an online setting.

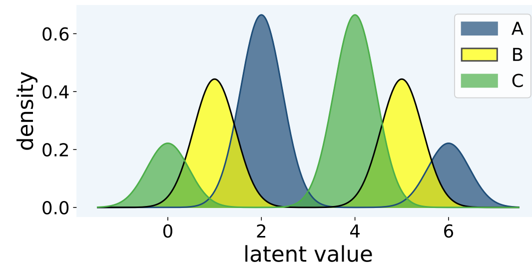

Figure 2: Three distributions that have intransitive behavior: $P(A > B) > 0.5, P(B > C) > 0.5, P(C > A) > 0.5$.

Intransitivity of $\xrightarrow{p}$***:*** Perhaps the most counter-intuitive property of $\xrightarrow{p}$ is its intransitivity. It is possible for the probability that event A precedes B to be high, the probability that B precedes C to be high, and yet the probability that C precedes A to also be high. It stems from the nature of how probabilities may behave [19]: the probability distributions may overlap in a way to show the cyclic behavior as seen[1] in Figure 2. As a result, the $\xrightarrow{p}$ relation, a probabilistic relation, may exhibit intransitivity, preventing direct construction of a fair order from pairwise relations. Fortunately, social choice theory extensively deals with ranking of objects where the pairwise comparison operator can be intransitive [20]. We reduce fair sequencing to the ranked voting problem in social choice theory to find an ordering of multiple events using the $\xrightarrow{p}$ relation. While intransitivity may still preclude the possibility of imposing a total order (due to *Condorcet cycles*, a phenomenon we discuss in §3.4), this transformation allows us to extract a partial order in spite of adverse clock uncertainty.

Online Sequencing: Online sequencing poses challenges beyond those encountered when ordering a fixed set of events. These challenges stem from the possibility that future events may need to be assigned the same or a lower rank than events which have already arrived at the sequencer. To resolve this, we extend our statistical model to the online setting as follows: before committing to the rank of an event, the sequencer identifies, via a statistical model, the range of timestamps for all clients that may require an equal or lower rank than the current event's timestamp; it then waits for any events that may fall within this range, and sequences them jointly with the current event to find a stable order.

We show that Tommy outperforms a Spanner TrueTime [21] based baseline, which represents each event timestamp as an uncertainty interval and orders events by sorting these intervals. Tommy produces orderings that are closer to the ideal fair order and, in some cases, exactly match it. To the best of our knowledge, Tommy is the first system to

[1]Each is a mixture of Gaussians, with standard deviation of 0.45. Centers are visible in the diagram. Known solution forms exist to calculate the shown probabilities.

demonstrate that probabilistic comparisons of noisy timestamps, combined with tools from social choice theory, can yield effective fair event ordering.

2 RELATED WORK

2.1 Financial Exchanges

Ordering in cloud-native exchanges. CloudEx [8] and Onyx [14, 15, 22] are recent proposals that aim to provide fairness in trading among market participants. Several market participants may send buy or sell commands to an exchange upon some sensitive event in the market. Exchanges often offer the guarantee that whoever asked first, would get their ask met first. For approximating this guarantee, CloudEx and Onyx approximate fair ordering of events generated by clients. Both systems assume tightly synchronized clocks across all participants and the exchange server. While such assumptions are practical in special circumstances, e.g., all VMs residing in a single data center using advanced clock synchronization [23], they fail to deliver desirable fairness in general settings. Generally, clock errors are large enough that two closely occurring events (e.g., microseconds apart) cannot be ordered correctly using the event timestamps from the local clocks of the respective clients. Tommy does not rely on tight clock synchronization; instead, it derives ordering decisions from the observed clock error distribution, which encodes more robust information for determining relative order. Tommy still requires some clock synchronization protocol (e.g., NTP [24]) so that it can build a distribution of clock corrections that is used by Tommy's statistical model for computing the fair (partial) order of events with the help of tools from social choice theory.

Ordering in on-prem exchanges. On-prem exchanges establish fair ordering by investing substantial engineering effort in bespoke infrastructure [25]. In particular, they ensure that all participants are connected to the exchange server using equalized data paths (e.g., same-length cables and low jitter switches). Such fine-grained engineering down to physical wire lengths is feasible in on-prem exchanges, where the entire infrastructure is purpose-built for trading. However, this approach does not generalize (e.g., to cloud environments or wide-area networks), where bespoke hardware solutions are impractical and incompatible with the needs of diverse stakeholders. Tommy, on the other hand, does not require any specialized infrastructure.

2.2 Departing from Arbitrary Order

Recent work, Pompe [26], observes that in blockchain systems, different participants have a direct stake in the specific ordering of events, and that it is therefore necessary to not be content with an arbitrary order of events produced by a sequencer. Pompe allows each replica node of a replicated state machine to provide hints about the desired order of some events. It draws on social choice theory to reframe ordering in the presence of Byzantine failures as a problem of preference aggregation under adversarial voters. In particular, it allows each participant to use physical clock timestamps as ordering indicators and collectively "vote" on a democratic ordering of events. It does not guarantee (as it is not its goal) that the final order of events will be fair in the sense that an earlier generated event will be ordered ahead of a later generated event.

Pompe's approach relies on the assumption that clocks at correct nodes are accurate and treats timestamps as reliable and definitive ordering indicators. This assumption simplifies the ordering problem and avoids pathological cases such as intransitivity of ordering indicators. In contrast, in Tommy, we argue that clock synchronization is inherently imperfect and that timestamps (from clients' local clocks) alone cannot fully determine precedence relationships between events. Tommy incorporates this imperfect clock synchronization into its statistical model for ordering.

3 BACKGROUND

3.1 Fair Ordering

Definition: We consider a set of events generated at multiple clients, where each event occurs at some global time at its originating client. A sequencer observes these events and produces a total order over them. The ordering is fair if, whenever one event occurs earlier than another in global time, it is ordered before the later one.

The definition characterizes an ideal notion of fairness that is generally impossible to realize: ordering events that occur arbitrarily close in time, but at different clients, would require resolving clients' clock errors below the fundamental uncertainty imposed by network delays, which is provably unattainable even with optimal synchronization algorithms [18]. Considering this, We relax the target and define *probabilistic fair ordering.*

Probabilistic Fair Ordering: If an event occurs earlier than another according to the global time, the sequencer orders the earlier event before the later one with high probability. There are no guarantees on how high the probability can be.

Our sequencer, Tommy, achieves probabilistic fair ordering. It assigns ranks to all the observed events, where it attempts to assign a higher rank to events that occurred later and lower ranks to events that occurred earlier. It may assign same ranks to some events if it is not confident about their relative order. In our experiments, we show Tommy outperforms baselines substantially.

Failure Model: We assume a fail-stop model. As we explain later, each client sends heartbeats to the sequencer periodically. Absence of heartbeats from a client for some period of time is a signal for the sequencer to exclude that client from consideration. As it is a standard mechanism deployed widely, we do not discuss it further. Moreover, an interesting model to explore in the future is Byzantine failures. However, for now we believe the problem is interesting and complex enough without assuming Byzantine failures.

3.2 Applications requiring fair ordering

Fair ordering is widely regarded as a fundamental requirement for many practical systems, including financial exchanges [8, 9, 14, 15, 27], online gaming [28], and ad auctions [7]. In such systems, fairness requires that clients have equal access to the exchange server: the order in which client writes are applied should reflect the order in which clients initiated those writes, rather than differences in network latency between clients and the server.

To achieve fairness, a straightforward approach is to rely on synchronized clocks across the exchange and all participants. Each participant adds a timestamp, via a trusted component, to their order, before sending it to the exchange. The exchange sorts incoming orders using the attached timestamps to achieve fair ordering. Indeed, several recent systems [8, 14, 15] adopt this design to build fair-access exchanges in cloud environments. However, the effectiveness of such approaches depends on near-perfect clock synchronization. Although recent advances have significantly improved clock synchronization [23, 29, 30], it remains unrealistic to generally assume *perfectly/tightly synchronized clocks.* In fact, classic results by Lundelius and Lynch [18] show that no algorithm can synchronize the clocks of n servers more closely than $U(1-1/n)$, where U denotes the uncertainty in message delay (i.e., the difference between the maximum and minimum message delays). Consequently, under high-frequency and high-concurrency workloads, even microsecond-scale clock skew can lead to ordering errors at the exchange. For example, Client 1 may generate an order 1 µs earlier than Client 2, yet the exchange, after receiving the orders from both clients, may incorrectly prioritize Client 2's order over Client 1's and may sell a scarce asset to Client 2 incorrectly inferring that the Client 2 generated the order first.

Motivated by these limitations, we depart from approaches that rely on near-perfect clock synchronization and instead pursue an approach that works with best-effort synchronized clocks, with the caveat that the the ordering would be fair probabilistically i.e., it outputs an order which it deems *likely to be fair* given the information it has but may make mistakes. In §8, we show a scenario where the probabilisitic nature of Tommy leads it to make more mistakes than a simple baseline solution, but generally, Tommy outperforms the baseline.

3.3 Clock Drift vs. Offset Considerations

Clock errors have two major components: drift and offset. Drift corresponds to rate mismatch between clocks while offset denotes the instantaneous mismatch between the values of two clocks. Most clock synchronization protocols can effectively correct for clock drift and reduce it to reasonable levels, whereas offset correction faces fundamental limitations. Drift can be reduced either by increasing synchronization frequency or by explicitly estimating clock frequency. For example, in NTP-style protocols, if clocks have fractional drift bounded by $\rho_{\max}$ and probes are exchanged every W seconds, the worst-case relative drift accumulated between two clocks (in the W second period) is bounded by

$$\delta_{\text{drift}} \leq 2\rho_{\max} W,$$

which decreases linearly as the probe interval W decreases. Similarly, protocols such as Chrony [31] estimate oscillator properties to infer clock frequency and continuously adjust clock readings to compensate for drift.

In contrast, offset uncertainty cannot always be reduced sufficiently. Lundelius and Lynch [18] show that in a network with one-way message delay uncertainty $U = d_{\max} - d_{\min}$, no clock synchronization algorithm can guarantee a maximum clock error smaller than

$$\delta_{\text{offset}} \geq U\left(1 - \frac{1}{n}\right),$$

where n is the number of participating clocks, $d_{\max}$ is the maximum possible delay and $d_{\min}$ is the minimum possible delay. This bound holds even in the absence of drift.

The best-case scenario (lowest offset error) occurs when $n = 2$, (excluding the trivial case of $n = 1$), for which the lower bound reduces to $\delta_{\text{offset}} \geq \frac{U}{2}$. Even in this minimal setting, offset uncertainty remains non-zero and independent of the probe frequency and in practice, significantly larger than the drift-induced error.

Concrete example. With conservative drift $\rho_{\max} = 20$ ppm and synchronization probes every $W = 20$ ms, the worst case drift induced misalignment is $\delta_{\text{drift}} \leq 2 \cdot 20 \times 10^{-6} \cdot 0.02 = 0.8$ µs. In contrast, with one way delay uncertainty $U = 40$ µs, even two clients incur an offset uncertainty of at least $\delta_{\text{offset}} \geq U/2 = 20$ µs. Thus, frequent probing reduces drift error below 1 µs, while offset uncertainty remains an order of magnitude larger and cannot be ignored at microsecond granularity.

Implication: Accordingly, we assume that deployed clock synchronization protocol adequately account for drift-related errors and makes it negligible w.r.t. the time granularity at which the application operates. And, we focus on reasoning about offset-related errors, the unavoidable component, in our fair ordering problem. In evaluation, we measure the impact of drift error on the fairness.

3.4 Ranking Under Uncertainty and Social Choice Theory

Given two events e_1 and e_2 with generation timestamps t_1 and t_2, respectively from the clients' local clocks, it may be incorrect to conclude that o_1 should be ordered ahead of o_2 solely because $t_1 < t_2$, because of the clock synchronization errors. Clock synchronization errors must be explicitly accounted for when determining the relative order of events. Motivated by this observation, we develop an approach that ranks events by jointly considering their generation timestamps and the probability distribution of clock corrections/offset adjustments.

A simple way to account for clock synchronization errors is to construct an uncertainty interval around each event timestamp, inspired by Spanner TrueTime [21]. And then events could be ordered by sorting these intervals i.e., events corresponding to non-overlapping intervals can get distinct ranks and those corresponding to the overlapping intervals can be given the same rank to account for high uncertainty. This is indeed our baseline ordering technique that we use to compare against Tommy. Although a good starting point, this approach is less helpful if the gaps between occurrence of events are small enough and the clock errors are comparatively larger as it may lead to most events having overlapped uncertainty intervals. In such a scenario, it becomes incumbent to zoom into the overlapping uncertainty intervals and attempt to assign distinct ranks to the events based on where the most *uncertainty mass* lies in the intervals. This is the precise motivation for our probabilistic fair ordering.

While the probabilistic ordering more accurately reflects the underlying uncertainty all the while enabling a useful sequencer, it also introduces cases where a total order of events cannot be established. A probabilistic relation to order two events can lead to cycles as such a relation is not necessarily transitive. For example, given three orders o_1, o_2, and o_3, it is possible that o_1 precedes o_2 with high probability, o_2 precedes o_3 with high probability, and yet o_3 precedes o_1 with high probability. In this case, pairwise comparisons fail to yield a globally consistent ordering. Such situations correspond to *Condorcet cycles* in social choice theory [20].

In a ranking problem in social choice theory, deciding the winner of an election based on pairwise preferences aggregated from an electorate (where each voter states which candidate they prefer among a pair of candidates) may lead to cyclic behavior i.e., majority may prefer candidate A over B, B over C and C over A, barring the selection of a clear winner. A winner in this problem is called a Condorcet winner and it is not guaranteed to exist. The diagnosis of this problem is that the pairwise preference relation used in ranking is not necessarily transitive, similar to the pairwise probabilistic ordering relation.

To address this challenge, social choice theory proposes the notion of the *Smith set* [32], which groups together all alternatives involved in Condorcet cycles such that every candidate in the Smith set beats every other candidate outside the set in a pairwise comparison, while the candidates within the set may or may not win over each other. Then the outcome of the election system is a set instead of single (Condorcet) winner.

Inspired by this concept, we design our sequencing algorithm to cluster events into Smith sets and enforce a deterministic ordering across sets. Within each Smith set, we intentionally refrain from imposing a strict order; instead, we delegate the choice of intra-set ordering to the application, allowing flexibility in how ties or near-ties are resolved, leading to a partial order instead of a total order produced by the sequencer.

4 TOMMY SEQUENCER DESIGN

As a clock synchronization protocol runs, it adjusts each client's clock periodically. These adjustments constitute the correction distributions for each client.

As an event e occurs at a client c, the client generates a timestamp, t_e, from its local clock to denote when the event occurred. The event timestamp, t_e, may not exactly represent the moment e occurred as there is some lag, however minute, between the occurrence of e and generating the t_e in any real-world system. We ignore this lag.

Clients convey the information about events to the sequencer by sending messages that contain event timestamps. A client c ensures that the message for t_e is not sent after the message for $t_{e'}$ if $t_e < t_{e'}$ and e, e' are generated at c. The sequencer uses the event timestamps as well as the clock correction distributions to assign ranks to the events that constitutes probabilistic fair ordering.

The sequencer uses a statistical model to find the probability of one event e being earlier than another event e' w.r.t. the global clock by only comparing the event timestamps t_e and $t_{e'}$. This is enabled by treating global time as a random variable. Once the pairwise probabilities for every two events are calculated, the sequencer uses a social choice theory based solution to deal with intransitivities and find a partial order of all events.

Likely-happened-before relation: The foundation of fair ordering is a pairwise relation, which we term likely-happened-before, denoted as $\xrightarrow{p}$, where p represents the probability that one event occurred before another. Tommy models global time at which the event occurred as a random variable: for an event e, global time of event occurrence, T_e is expressed as a linear combination of the event timestamp, t_e, from the respective client's local clock and the client's clock corrections distribution. By comparing realizations of these random variables for two events, the sequencer computes the probability that one event precedes the other. This probability directly defines the $\xrightarrow{p}$ relation.

Constructing the $\xrightarrow{p}$ pairwise relation is the key gadget that enables Tommy's fair ordering. In contrast, attempting to eliminate all synchronization errors from each client's local clocks to achieve fair ordering (by simply ordering events based on sorted timestamps) poses a substantially more difficult problem than probabilistically comparing two global timestamps.

Ordering multiple events using the $\xrightarrow{p}$ relation: Unlike the classical *happens-before* ($\rightarrow$) relation, the $\xrightarrow{p}$ is not a transitive relation. So obtaining an order of more than two events using the pairwise $\xrightarrow{p}$ relation is a challenge. We map the problem of finding order of events using the pairwise relations to a similar problem in the social choice theory literature: ranking candidates in an election using pairwise preferences (i.e., Candidate A is preferred over candidate B) obtained from an electorate. The reduction of ordering of events problem to a well-known ranking problem lends us mature tools that explicitly accounts for possible intransitivity in the pairwise preferences/ordering.

Online Sequencing: As a sequencer receives events from clients, it needs a mechanism to determine when the current ordered set of events is stable enough to be emitted (to any downstream application). We consider a set of events stable if the probability that any future event would arrive and be ordered ahead of (or equivalently, interleaved before) any event in the set is sufficiently low. Furthermore, achieving high performance as events arrive in a stream requires processing incoming events with minimal latency, which in turn demands efficient implementations of both the statistical model and the social choice theory–based ranking mechanism. We propose a *probability-sketch* based efficient algorithm for finding probability of an event to have occurred before another. Our algorithm leverages the observation that clock-correction distribution of a client does not change significantly at short timescales. In the subsequent sections, we expand on how the pairwise probability is calculated to construct the $\xrightarrow{p}$ relation (§5), how the relation is used to get an ordering of several events (§6) and how the sequencer operates in an online fashion (§7).

5 CONSTRUCTING LIKELY-HAPPENED BEFORE, $\xrightarrow{p}$, RELATION

An event i with event timestamp t_i from the local clock of the respective client, $c(i)$, has the global timestamp T_i. Under ideally synchronized clocks, $T_i = t_i$. However, in the real-world, every clock synchronization protocol is imperfect. We represent the global timestamp as: $T_i = t_i + \theta_i$, where θ_i denotes the client's clock correction relative to the sequencer's clock (the global clock) at the instant event i is generated. The correction θ_i is unknown but is modeled as a random variable following a probability distribution f_{θ_i}.

Clock correction distributions may differ across clients due to heterogeneous synchronization conditions, such as temperature variation across a data center or asymmetric network latency [23, 33]. Each client estimates its own correction distribution by collecting clock synchronization probes. A clock synchronization probe is an adjustment made to a client's clock by a clock synchronization protocol. A sequencer can observe t_i for each event and f_{θ_i} for each client.

Comparing two global timestamps: It is impossible to compute T_i exactly but we can compare two global timestamps T_i and T_j by only observing t_i and t_j using a probabilistic analysis that assumes the knowledge of clock correction distributions f_{θ_i} and f_{θ_j}.

We analyze the probability that one event precedes another. This probability is called the *preceding-probability*:

$$\mathbb{P}(T_i < T_j \mid t_i, t_j) = \mathbb{P}(t_i + \theta_i < t_j + \theta_j).$$

Rearranging: $\mathbb{P}(T_i < T_j \mid t_i, t_j) = \mathbb{P}(\theta_i - \theta_j < t_j - t_i)$. Since θ_i and θ_j are random variables, their difference follows a new distribution:

$$\Delta\theta = \theta_i - \theta_j \sim f_{\Delta\theta}.$$

Then the preceding-probability is given by:

$$\mathbb{P}(T_i < T_j \mid t_i, t_j) = f_{\Delta\theta}(t_j - t_i) = \int_{-\infty}^{t_j - t_i} f_{\Delta\theta} d\Delta.$$

If f_{θ_i}, f_{θ_j} are modeled as any known distributions e.g., Gaussian, then we may get a known solution form for the above integral. We have explored modeling f_{θ_i} and f_{θ_j} using such distributions, by fitting these distributions to clock synchronization probes collected in our experiments. However, we find that imposing such modeling assumptions consistently leads to worse fairness compared to making no distributional assumption at all. Therefore, we make no assumption about the shapes of f_{θ_i}, f_{θ_j}. We evaluate the above integral empirically using the following estimator:

$$\hat{f}_{\Delta\theta}(t_j - t_i) \;=\; \frac{1}{|f_{\theta_i}|\,|f_{\theta_j}|} \sum_{\theta_i \in f_{\theta_i}} \sum_{\theta_j \in f_{\theta_j}} \mathbf{1}\big[\theta_i - \theta_j < t_j - t_i\big]\,.$$

The estimator makes an independence assumption between f_{θ_i} and f_{θ_j} The assumption might not be always true but this is a simplification choice we make and may lead to sub-optimal ordering. In evaluation, we note Tommy outperforms existing mechanisms despite our simplification

Algorithm 1: Pairwise Ordering Probability

Input: Local timestamps x, y;
Correction probes $C_x = \{c_x^1, \ldots, c_x^{n_x}\}$; probes carry synchronization protocol's adjustments to the clock
Correction probes $C_y = \{c_y^1, \ldots, c_y^{n_y}\}$
Output: $p = \Pr(x + c_x < y + c_y)$

$\tau \leftarrow y - x$
count $\leftarrow 0$
total $\leftarrow n_x \cdot n_y$
foreach $c_x \in C_x$ **do**
  foreach $c_y \in C_y$ **do**
    if $c_x - c_y < \tau$ **then**
      count $\leftarrow$ *count* $+ 1$
return *count*/*total*

Algorithm 2: Precompute Difference Distributions

Input: Clock corrections probes sequences for all clients $\{C_1, \ldots, C_n\}$, where $C_i = \{c_i^1, \ldots, c_i^{m_i}\}$ is the sequence for one client
Output: Sorted difference lists $\mathcal{D}_{ij}$ for all i, j

for $i \leftarrow 1$ **to** n **do**
  for $j \leftarrow 1$ **to** n **do**
    $\mathcal{D}_{ij} \leftarrow \emptyset$
    foreach $c_i \in C_i$ **do**
      foreach $c_j \in C_j$ **do**
        Append $(c_i - c_j)$ to $\mathcal{D}_{ij}$
    Sort $\mathcal{D}_{ij}$
return $\{\mathcal{D}_{ij}\}_{i,j=1}^{n}$

Algorithm 3: Pairwise Ordering Query

Input: (1) Events e_i^x, e_j^y where x, y are local timestamps from clients i, j. (2) $\mathcal{D}_{ij}$
Output: $p_{ij} = \Pr(x + c_i < y + c_j)$

$\tau \leftarrow y - x$
$k \leftarrow$ UpperBound$(\mathcal{D}_{ij}, \tau)$; `// Count values <` τ
return $k/|\mathcal{D}_{ij}|$

assumption. Algorithm 1 implements the above estimator to find the pairwise preceding probability for two events.

Likely-happened-before and intransitivity: Once we can find preceding-probability for two events, this is the basis of the *likely-happened-before*, $\xrightarrow{p}$, relationship. It is a pairwise relation where $i \xrightarrow{p} j$ denotes that event i happened before event j with probability p. This is the foundation for the fair ordering achieved by Tommy.

If $\xrightarrow{p}$ was a transitive relation then finding a total order for a set of events would be trivial. We define transitivity as: If $i \xrightarrow{p>0.5} j$ and $j \xrightarrow{p>0.5} k$ then we must have $i \xrightarrow{p>0.5} k$. However, the transitivity may not always hold. In case it was to hold, then fair ordering would simply be a topological ordering of the graph where nodes are events and an edge from i to j represents $i \xrightarrow{p>0.5} j$.

In some special cases, the transitivity for $\xrightarrow{p}$ may hold. For example, Appendix A shows that if clock correction distributions are Gaussian, then the transitivity provably holds. We also implement the proof in Lean. The existence of transitivity depends on the behavior of probabilities and it generally does not hold [19]. Finding the necessary and sufficient conditions under which transitivity may hold is left for future work.

Due to the intransitivity, $\xrightarrow{p}$ may not always allow a total order of events. In our ns-3 simulations with a data-center wide background workload, we see that such intransitive behavior is indeed manifested while attempting to order events based on $\xrightarrow{p}$, thereby preventing us from achieving a total order of events. Despite the existence of intransitivity, in §6, we will discuss how a partial order of events can still be achieved by using lessons from social choice theory.

Efficient events processing: Pairwise preceding probabilities are a core primitive used repeatedly by the fair ordering procedures. As we discuss next, in Algorithm 4, constructing the matrix P requires computing pairwise preceding probabilities for all $O(n^2)$ pairs of events. Using Algorithm 1 to compute the preceding probability for a single pair of messages incurs quadratic time complexity with respect to the number of samples in the clock correction distributions.

We observe that clock correction distributions remain stable over time and only change if the background workload shifts significantly or the underlying network connectivity changes, i.e., some link failure leading to rerouting of packets through a path which may have different characteristics than the abandoned path. Assuming the distributions stay stable over timescale of M minutes, the sequencer can periodically pre-compute *difference distributions* using Algorithm 2 after every M minutes. Once these difference distributions are available, the pairwise preceding probability for any pair of events can be computed using Algorithm 3 in logarithmic time as opposed to the quadratic time with respect to the number of samples in the clock correction distributions.

One disadvantage of pre-computing difference distributions is that if the difference distributions become outdated, the resultant values of preceding probabilities may become incorrect leading to wrong ordering. The distributions may become outdated if the timescale of refreshing the distributions is too large which is relative to the network environment in which Tommy is deployed. We excuse ourselves from further discussion of this problem but may require more research if Tommy is to be deployed in production.

Algorithm 4: Probabilistic Fair Ordering

Input: (1) Event log $E = \{e_1^t, \ldots, e_n^{t'}\}$ where each event $e_n^{t'}$ was generated by client n at local timestamp t';
(2) $\{\mathcal{D}_{ij}\}_{i,j=1}^n$ (generated by Algorithm 2)
Output: Ordered batches of events $\mathcal{B}_1, \mathcal{B}_2, \ldots$

```
// Generate all to all matrix for pairwise
   probabilities P_ij for all events
```

$P \leftarrow$ PairwiseProbMatrix$(E, \{\mathcal{D}_{ij}\}_{i,j=1}^n)$
$G \leftarrow$ directed graph with vertices $\{1, \ldots, n\}$
for $i \leftarrow 1$ **to** n **do**
    for $j \leftarrow 1$ **to** n **do**
        if $P_{ij} > \frac{1}{2}$ **then**
            add directed edge $i \rightarrow j$ to G
Compute strongly connected components (SCCs) of G
Collapse SCCs into a DAG G'
Compute a topological ordering of G'
return SCCs in topological order

6 ACHIEVING PARTIAL ORDER

Although $\xrightarrow{p}$ is helpful in ordering two events, it is not a general ordering relation because of its intransitivity. This problem is analogous to a problem in social choice theory literature called *Condorcet Paradox* [20] briefed next.

In a ranking problem in the social choice theory, given a set of pairwise preferences of election candidates, the goal is to find the election winner, called a *Condorcet winner*. A pairwise preference is a preference of one candidate between two candidates and is analogous to the $\xrightarrow{p}$ relation where both are intransitive. A *Condorcet* winner is a candidate who would receive the support of more than half of the electorate in a one-on-one race against any one of their opponents. The *Condorcet Paradox* is that such a winner may not exist. However, a generalization of *Condorcet* winner called a *Smith Set* is guaranteed to always exist.

A Smith Set is the smallest set of candidates that are pairwise unbeaten by every candidate outside of it. We extend the idea of finding Smith Set to get a partial order of events: (i) find a smith set from the events, (ii) assign the same rank to all events in that smith set, (iii) remove those events from the set of events and, (iv) repeat while incrementing the assigned rank and until all events have been ranked/ordered.

The above solution provides a partial order as some events are not ranked w.r.t each other, i.e., they are assigned the same rank. We forsake achieving a total order and settle for a partial order. An application can have its own rules for extending the partial ordering to a total order, an already exercised option where applications decide how to deal with concurrent events when the $\rightarrow$ relation leads to a partial order. The sequencer releases batches (smith sets) of events at a time to any downstream application. An application can decide for itself how to deal with tied ranks. For example, a financial exchange may try to randomly find a bidder from a batch of bidders to sell an asset to or divide an asset proportionally to all bidders in a batch.

The above mentioned repeated smith set extraction can be implemented as a graph problem. The sequencer creates a graph where each node is an event. An edge exists between two nodes i, j if $i \xrightarrow{p>0.5} j$. Then the sequencer runs Tarjan's Algorithm [34] on the resultant graph, to find all the Strongly Connected Components (SCC) of the graph. Finally, it finds a topological ordering of the SCCs. Each SCC is a smith set and they are emitted from the sequencer in the order of their topological order. Algorithm 4 implements the above.

7 ONLINE SEQUENCING

Our discussion so far has assumed that the sequencer needs to order a set of available events. However, in reality, the sequencer needs to operate online, ordering events as they arrive as a stream. For *online sequencing*, we need a way to answer the following question: should the sequencer order and emit the current events (to any downstream application) or wait for further events? The waiting would be needed if a new arriving event has a lower rank than the already present events. In such scenarios, skipping the wait may lead to a sequencer output that disagrees with the fair order of events.

Once the sequencer knows all new events will only need a higher rank than the already arrived events, it can order the existing events and deem it stable for emitting from the sequencer. In the following, we discuss how this stability condition can be checked.

Consider an event m from client c generated at timestamp t. For this event, the sequencer computes a *stable timestamp* for every other client. A stable timestamp t' for client c' implies that any event m' generated by c' at or after t' has a very high probability of being assigned a rank strictly higher than that of m. Therefore, if the sequencer has received all events with timestamps smaller than the respective clients' stable timestamps, it can safely order and emit the events.

To enable this determination, each client periodically sends heartbeat events to the sequencer. A heartbeat generated at timestamp t and received by the sequencer guarantees that all events generated by the sending client at timestamps $t' < t$ have already been received by the sequencer.

Using heartbeats, the sequencer can check whether all events with timestamps earlier than the computed stable timestamps have arrived. If additional events with timestamps smaller than the current stable timestamps arrive, the sequencer must re-compute the stable timestamps accounting for the new events. Algorithm 5 implements the online sequencing procedure that enforces the stability condition.

Algorithm 5: Online Sequencing

```
Input: Number of clients N;
State: buffer ← ∅ (received but unordered events);
watermark[c] ← −∞ for all c ∈ {0, . . . , N − 1};
Function OnEventArrival(e):
    buffer ← buffer ∪ {e};
    AttemptOrder();
Function OnHeartbeat(c, t):
    watermark[c] ← max(watermark[c], t);
    AttemptOrder();
Function AttemptOrder():
    if buffer = ∅ then
        return;
    batches ← OrderBuffer(buffer);
    // batches is a list of batches/SCCs
       generated by Algorithm 4
    (t_F, x_F) ← Frontier(batches);
    // Choose frontier event in last batch
       with maximum local time t_F and its
       client x_F
    stable_until[·] ← StableTimes(t_F, x_F);
    for c ← 0 to N − 1 do
        if watermark[c] < stable_until[c] then
            return ;  // Not safe yet; wait for
              more events/heartbeats
    Emit(batches) ;           // Safe to commit
    buffer ← ∅;
Function StableTimes(t_x, x):
    // For each client c, find the smallest t
       such that Pr(t_x + c_x < t + c_c) ≈ 1
    // (via per-client binary search on
       future timestamp t)
    return stable_until[·];
```

We show in §8 that with large clock correction distributions, it may take longer to reach the stability condition which, in turn, delays emitting ordered events from the sequencer. Hence, any production realization of Tommy may add sanity checks around unusually high clock sync. errors that lead to large correction distributions and may bar the corresponding clients from partaking in the sequencing for the sake of liveness.

8 EVALUATION

For evaluating Tommy, we require ground truth, namely the true temporal order in which events occur, against which we can compare the order produced by Tommy. However, obtaining ground truth in real-world systems is fundamentally challenging, as it would require perfectly synchronized clocks, which are impossible to realize.

Difficulty of obtaining ground-truth: One might attempt to approximate ground truth by using clocks that are significantly better synchronized than those available to Tommy. However, this approach is insufficient on its own. Even if two clocks with different synchronization accuracies are available, the variance (across clients) in latency between reading the two clocks can introduce additional uncertainty, thereby obscuring the true event order. Only in an idealized setting where both clocks could be accessed simultaneously would it be possible to treat one clock as ground truth while using the other for Tommy.

As a concrete example, consider two clients, A and B, each reading two clocks in sequence. Client A first reads `clock1` at time t_1 and then reads `clock2` at time t_4. Client B reads `clock1` at time t_2 followed by `clock2` at time t_3. Assume that `clock1` timestamps the event occurrence, while `clock2` is a perfectly synchronized reference clock, and that the observed times satisfy $t_1 < t_2 < t_3 < t_4$.

Under `clock1`, the event at client A precedes the event at client B. However, under the perfectly synchronized `clock2`, the event at client B precedes the event at client A. This discrepancy arises from asymmetric clock-access latencies, showing how even a perfectly synchronized reference clock may fail to provide ground truth in practice. This is problematic for Tommy evaluation as most of its benefit becomes apparent when inter-event-occurrence duration is small e.g., a few microseconds. But a few microseconds latency of variance in accessing two clocks can render the ground truth useless. Consequently, establishing reliable ground truth for event ordering remains non-trivial, which motivates our design of simulation-based methodology described below.

Simulator-based setup: We use the ns-3 simulator to instantiate multiple clients and a server acting as the sequencer. Event generation at clients is explicitly controlled, allowing us to prescribe a ground-truth order of events. For example, an input sequence c_i^1, c_j^2, c_k^3 specifies that client i generates the first event, followed by client j, and then client k. The inter-event durations are configurable.

When an event occurs at a client, the client records a local timestamp using a simulated virtual clock with configurable offset and drift. These timestamps are provided as inputs to the sequencer, which produces an ordered sequence of events. We evaluate Tommy by comparing the sequencer's output order against the prescribed input sequence.

Each client and the server has a simulated virtual clock. The offsets and drifts for each clock are selected from a normal distribution whose parameters are configurable. Clients run NTP protocol to synchronize their clocks with the server's clock which is assumed to be the global time. The clock sync. accuracy depends on the latency experienced by the packets containing the synchronization probes. To make the synchronization realistic, we configure the latency distribution to what is expected in a real data center.

Realistic latency distribution: In a separate ns-3 simulation, we simulate a fat-tree data center topology from Homa [35, 36]: 144 hosts, 9 ToRs, 4 spines where each ToR downlink is connected to mutually exclusive set of 16 hosts and each ToR's uplink is connected all spines. ToR downlinks' have a capacity of 10Gbps each where the uplinks' have a capacity of 40 Gbps each. More details about the workload in Appendix B. We gather latencies experienced by the packets in this simulation. These latencies are used in our Tommy simulation so that clock sync. probes can experience near-realistic conditions.

Baseline: We develop a baseline to compare against Tommy. It is inspired by Spanner's TrueTime API [21]. Each event timestamp is represented with an uncertainty interval around it i.e., for a timestamp t, we represent it as $[t - ErrorBound, t + ErrorBound]$. In the paper [21], the $ErrorBound$ is set to be a limit that the clock error generally will not cross and is highly dependent on the production environment. We infer the bound from the gathered clock sync. probes. We find the standard deviation, σ, from the gathered probes and set the bound to be 3σ. Once all event timestamps are represented as uncertainty intervals, we sort the intervals based on the starting point of intervals. The sorted intervals directly represent the order of the corresponding events while assigning the same order/rank to events if their intervals overlap. The above mechanism helps finding the order of a given set of events, while the online extension uses the same protocol presented in Algorithm 5.

Configuration: We use the following configuration unless specified otherwise. We have 100 clients with IDs: $c_1, c_2, ..., c_{100}$, generating events in the order of their client ID. Total 200 events are generated per experiment, while we perform 5 experiments and average out the fairness. The clock sync probes' frequency is set to 10 probes per second. The offset for each client's clock is taken from normal distribution with mean of 0 and standard deviation of 1 s.

Evaluation questions: We focus on the following aspects:

(1) How does fairness achieved by Tommy compare to that achieved by TrueTime-baseline?
(2) How do the different characteristics of an application (e.g., inter-messages delays, number of messages/clients etc.) impact the fairness achieved by Tommy?
(3) How is fairness impacted by the network latencies?
(4) How does clock sync protocol affect the fairness?
(5) Dive into several components of Tommy.

Comparison Metric: Rank Agreement Score (RAS) is proposed to quantify fairness given a ground truth ordered sequence and an ordered sequence output by Tommy or the baseline. RAS is computed as follows: for each correctly ordered pair of events, score increases by 1, for each incorrectly ordered pair, the score decreases by 1 while a pair of events left unordered w.r.t. each other adds 0 to the score. The resultant score is normalized by the total number of pairs of events. While this metric is helpful in most scenarios, it has some pitfalls we elaborate further while explaining the evaluation of Tommy. Higher RAS is better.

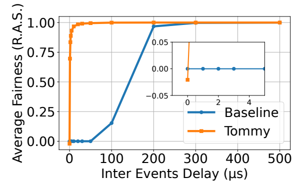

Figure 3: Tommy mostly achieves better fairness than the baseline.

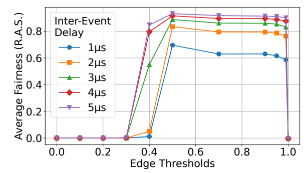

Figure 4: Optimal point for fairness arrives at edge threshold of 0.5.

8.1 Tommy vs. TrueTime Baseline

To show effectiveness of Tommy in fairly ordering events, we compare it against the baseline discussed above.

Figure 3 shows Tommy mostly achieves higher fairness compared to the baseline as we sweep the inter-event delay from 0 to 500 µs. At higher delays, ordering the events is easier as the uncertainty by clocks is smaller compared to the gap between event timestamps. Both Tommy and the TrueTime baseline achieves perfect fairness with delay=500 µs. Towards lower delays, Tommy establishes its effectiveness over the baseline. However, the baseline is a deterministic mechanism and may leave events unordered if it is uncertain achieving fairness closer or equal to 0. On the other hand, Tommy, as a probabilistic mechanism, may attempt to order events even under high uncertainty and make mistakes, leading to negative fairness in some scenarios; baseline's fairness never goes below 0. That's the reason for baseline achieving higher fairness than Tommy in Figure 3 with inter-event delay equal to 0. As clock's errors increase and the inter-event delay decreases, Tommy may make more ordering mistakes than the baseline, but such scenarios would be out of the operational regions for the most systems. We do one experiment with pathological configuration i.e., setting clock drift rate to be extremely high (100 ppm) and see that Tommy can achieve $\approx$ 18% lower fairness (RAS) than the baseline.

8.2 Properties Of An Application And The Impact on Fairness

Several properties of an application e.g., how many clients are involved, how many events occur per second, the inter-event delay can have impact on the fairness achieved by Tommy. We already show the impact of inter-event delay on

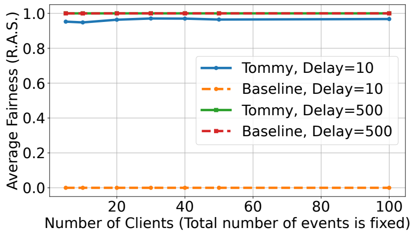

Figure 5: Number of clients (with no change in total events) have no effect on fairness.

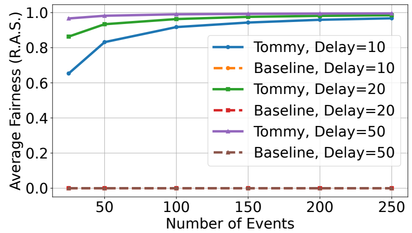

Figure 6: Fairness with Tommy is higher if ordering more events, an artifact of the fairness metric.

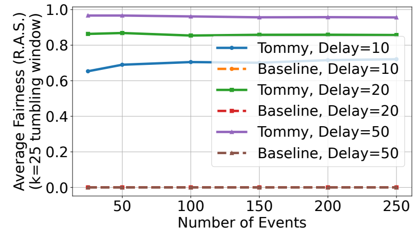

Figure 7: Fairness (over a constant sized window of events) stays constant as total number of events increases.

the fairness in §8.1. Here, we discuss the impact of number of clients and events, as well as a pitfall of the RAS metric.

Changing the number of clients: For a fixed number of events, evenly distributed among clients, changing the number of clients has no impact on RAS in our simulations. Figure 5 shows how the RAS remains almost constant as we sweep the number of clients. The effect stays consistent regardless of the inter-event delays (10 µs and 500 µs are shown in the figure). In these experiments, the extra knowledge, that the events occurring at the same client can be ordered using the happened-before relation, is not utilized as otherwise the ordering may improve as the number of clients is made small and the number of events stays constant.

Changing the number of events: We fix the number of clients to 25 and sweep the number of events from 25 to 250. Figure 6 shows that Tommy fairness improves as the number of events increase. This is a surprising result and upon further investigation, we realize it is an artifact of our RAS metric. As the number of events increase, a later event is ordered correctly with a large number of older events as the inter-event delay becomes high. Due to this, the number of correctly ordered pairs increases, so RAS increases. To isolate fairness from this effect, we only measure fairness for events that occur close together, i.e., we use a window (=25 events) and compute RAS for events only in that window. Figure 7 shows that increasing the number of events has no significant impact on the fairness computed over a constant sized window, which is an expected behavior.

Fairness is not directly related to the number of clients or events; rather, it depends on the inter-event delay, as shown previously. If changing the number of clients/events alters inter-event delay, the resulting impact on fairness will manifest. However, in the experiments above, we control for the inter-event delay, regardless of the number of clients/events.

8.3 Network Latencies Impact on Fairness

Network latencies can impact the clock synchronization probes as well as the messages that convey the event timestamps to the sequencer. Sequencer ensures that latencies of

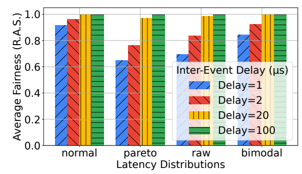

Figure 8: Latency models impact fairness.

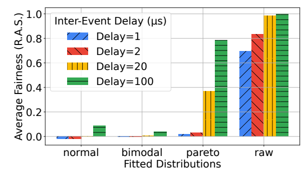

Figure 9: Fitting distributions not better.

messages carrying the event timestamps have no effect on fairness as the sequencer only progresses once it has seen certain messages from *every* client so the fairness calculations are idempotent w.r.t how much time certain messages took to arrive.

The impact of network latencies on clock synchronization probes, however, is noticeable as it dictates the clock synchronization accuracy and the clock correction distributions. First, for a sanity check, we feed several random latency distributions to our ns-3 Tommy simulation and study the impact on fairness. Figure 8 shows the fairness varies across multiple latency distributions. Second, as we collect latencies of packets from the fat-tree data center simulation, we fit a distribution to that data and then feed the learnt distribution to our Tommy simulation. Figure 9 presents the fairness achieved by not fitting any distribution and using the obtained histogram of latencies as-is achieves the highest fairness compared to fitting a normal, bimodal or even a pareto distribution. This establishes the sensitivity of Tommy to the latencies experienced by synchronization probes.

8.4 Effect of Synchronization Probes' Frequency

Clock synchronization probes' frequency directly impacts the fidelity of clock correction distributions estimated by clients. These distributions in turn directly impact the fairness achieved by Tommy. Figure 10 shows the length of the clock sync. interval, in our implementation of NTP protocol which synchronizes the virtual clock in our ns-3 simulation, is inversely proportional to the fairness achieved by Tommy.

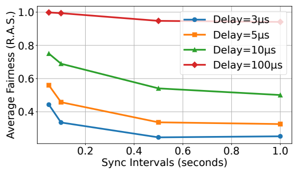

Figure 10: Clock sync. interval impact fairness.

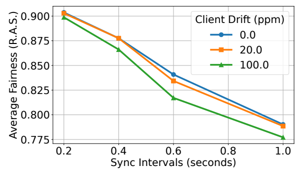

Figure 11: Higher drift lowers the fairness.

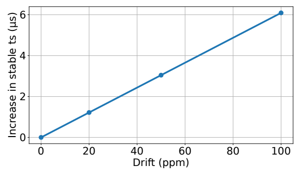

Figure 12: Future stable ts enlarge error grows.

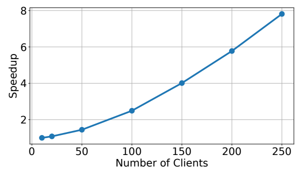

Figure 13: Pre-computing achieves speed up.

As one of our assumption of a deployed clock synchronization protocol is that it is adequately addressing the clock drift. Figure 11 uses the inter-event delay of 15 μs, and it shows how higher drift overall leads to lower fairness, while the synchronization intervals' effect also persists.

8.5 Dive into Tommy components

Effect of edge threshold: In §6, we only consider an edge from i to j if $i \xrightarrow{p>0.5} j$. The threshold 0.5 was chosen because it indicates that it is more likely that i happened before j than j before i. However, increasing the value of this threshold may provide more confidence in the ordering as we only consider edges which are much more likely to be true. But on the other hand, setting this threshold high bars several pairwise relations providing less freedom for Tommy to order the events. Figure 4 shows that as the edge threshold increases beyond 0.5, the fairness does not increase, but a slight decrease occurs. Further, setting threshold to lower than 0.5, sharply decreases the achieved fairness as Tommy stays indifferent about several events.

Stability Condition: As clock errors increase, the correction distributions widen. Larger distributions in turn lead to larger future timestamps that needs to be considered with a current event for it to be considered stable in an online setting. In Figure 12, we perform an experiment where we increase the clock error (drift) and study the increase in the future timestamps calculated for stability condition (Algorithm 5), compared to the case with zero clock drift. Figure 12 shows that the clock error and the increment in stable timestamps are directly proportional.

Benefit of pre-computing difference distributions: A main optimization is to pre-compute corrections-difference distributions for each pair of clients as it significantly improves the time it takes finding the pairwise preceding probability for events. Figure 13 shows as the number of clients increases, the speedup (=time taken without optimization / time taken with optimization) increases. In this experiment, each client generates 10 events in the same round robin fashion as described in the evaluation setup previously.

9 LIMITATIONS AND FUTURE WORK

Tommy has limitations stemming from modeling assumptions and evaluation scope.

Event definition: We treat an event as occurring when a client reads its local timestamp. In practice, application level actions may precede this read by variable delays, introducing additional noise that is not captured by our model.

Modeling assumptions: Our model assumes independence and short term stability of clock correction distributions. Correlated errors across clients or abrupt network changes may violate these assumptions and degrade ordering accuracy. Addressing such effects requires more complex models and adaptive mechanisms.

Evaluation scope: Our results are based on ns-3 simulations. While this enables controlled experiments and access to ground truth, real world deployments may exhibit additional sources of variability that we leave to future work.

Future Work: There are several promising directions for future work building on Tommy: (i) Strengthening the failure model. Tommy currently assumes fail stop clients. Extending the design to tolerate Byzantine clients would broaden its applicability, especially in open or adversarial environments. (ii) Scaling the sequencer. While this paper focuses on a single logical sequencer, scaling fair ordering to larger number of clients and larger workloads is a natural extension. (iii) A real-world deployment would allow deeper study of clock correction stability, correlation effects, and end to end application level fairness. It would also enable exploration of adaptive mechanisms that dynamically adjust sequencing behavior in response to changing network conditions.

10 CONCLUSION

We present Tommy, a sequencer that achieves probabilistic fair ordering of events in the presence of inevitable clock synchronization errors. Instead of attempting to eliminate timestamp uncertainty, Tommy embraces it by modeling clock corrections and using probabilistic comparisons to order events fairly. By reducing fair ordering to a ranking problem with intransitive preferences, Tommy leverages social choice theory to produce meaningful partial ordering of events. Our evaluation demonstrates that Tommy achieves better fairness than existing baselines, highlighting the feasibility of fair sequencing without relying on tightly synchronized clocks. This work does not raise any ethical concerns.

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A TRANSIVITY HOLDS FOR GAUSSIAN DISTRIBUTIONS

PROPOSITION 1. *Let X, Y, Z be independent normal random variables*

$$X \sim \mathcal{N}(\mu_X, \sigma_X^2), \qquad Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2), \qquad Z \sim \mathcal{N}(\mu_Z, \sigma_Z^2).$$

Define the preference relation

$$X \succ Y \iff \Pr[X > Y] > \tfrac{1}{2}.$$

Then $\succ$ is transitive: if $X \succ Y$ and $Y \succ Z$, we necessarily have $X \succ Z$.

PROOF. For any two independent Gaussian variables $A \sim \mathcal{N}(\mu_A, \sigma_A^2)$ and $B \sim \mathcal{N}(\mu_B, \sigma_B^2)$, the difference $A - B$ is Gaussian with

$$A - B \ \sim\ \mathcal{N}\big(\mu_A - \mu_B,\ \sigma_A^2 + \sigma_B^2\big).$$

Hence

$$\Pr[A > B] = \Pr[A - B > 0] = \Phi\Big(\frac{\mu_A - \mu_B}{\sqrt{\sigma_A^2 + \sigma_B^2}}\Big),$$

where Φ is the standard–normal CDF. Now:

$$\Pr[A > B] > \tfrac{1}{2} \iff \Phi\Big(\frac{\mu_A - \mu_B}{\sqrt{\sigma_A^2 + \sigma_B^2}}\Big) > \tfrac{1}{2}. \tag{1}$$

As $\Phi(0) = \frac{1}{2}$, so:

$$\Phi\Big(\frac{\mu_A - \mu_B}{\sqrt{\sigma_A^2 + \sigma_B^2}}\Big) > \Phi(0).$$

Because Φ is a strictly increasing function,

$$\Phi\left(\frac{\mu_A - \mu_B}{\sqrt{\sigma_A^2 + \sigma_B^2}}\right) > \Phi(0) \iff \frac{\mu_A - \mu_B}{\sqrt{\sigma_A^2 + \sigma_B^2}} > 0.$$

As the denominator $\sqrt{\sigma_A^2 + \sigma_B^2}$ cannot be negative,

$$\Pr[A > B] > \tfrac{1}{2} \iff \mu_A - \mu_B > 0 \iff \mu_A > \mu_B. \tag{2}$$

Thus our preference rule depends *only* on the means.

Now, suppose $X \succ Y$ and $Y \succ Z$. This implies

$$\mu_X > \mu_Y \quad \text{and} \quad \mu_Y > \mu_Z,$$

which together give $\mu_X > \mu_Z$ because means (i.e., real numbers) are transitive. Applying eq. 2 to $\mu_X > \mu_Z$, yields $X \succ Z$. □

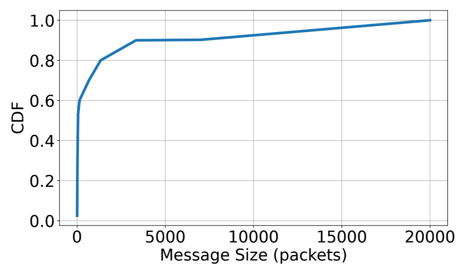

Figure 14: DCN workload from Homa [35, 36]

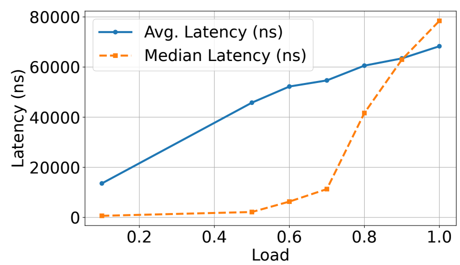

Figure 15: Characterizing the setup used for simulations.

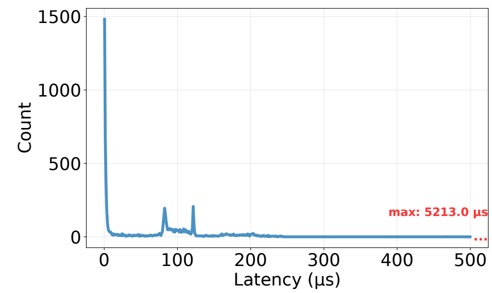

Figure 16: Histogram of latencies experienced by packets (load=1.0)

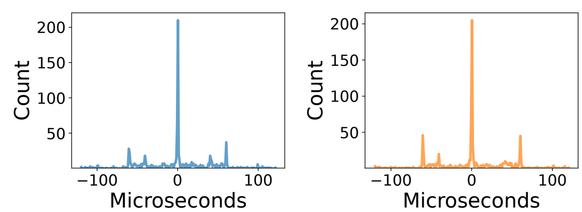

Figure 17: Clock correction distributions for two clients

B DATA-CENTER WORKLOAD SETUP

Each client has a 10Gbps uplink to a ToR. Clients are seeded with a message-size distribution from Homa [35], also shown in Figure 14. Clients can be configured to emit a load of certain ratio for example 0.1 denotes 10% of the uplink capacity is exhausted. TCP Cubic is used. Figure 15 shows the latencies of packets in this data-center simulation as we configure clients to several load ratios. The simulation ran for 0.05s of data-center workload duration (the time taken by the simulator was much longer) for each load ratio. Figure 16 shows the histogram of latencies with load ratio of 1.0, showing multiple modes and a long tail. When running Tommy simulation with this latency data, the clock correction distributions obtained are shown in Figure 17 (for two clients). These correction distributions also exhibit multiple modes.

C TRANSITIVITY HOLDS FOR GAUSSIAN DISTRIBUTIONS AT ANY FIXED THRESHOLD

Proposition 2. *Let X, Y, Z be independent normal random variables*

$$X \sim \mathcal{N}(\mu_X, \sigma_X^2), \qquad Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2), \qquad Z \sim \mathcal{N}(\mu_Z, \sigma_Z^2).$$

Fix any $p \in (\frac{1}{2}, 1)$ and define the preference relation

$$X \succ_p Y \iff \Pr[X > Y] > p.$$

Then $\succ_p$ is transitive: if $X \succ_p Y$ and $Y \succ_p Z$, we necessarily have $X \succ_p Z$.

Proof. For any two independent Gaussian variables $A \sim \mathcal{N}(\mu_A, \sigma_A^2)$ and $B \sim \mathcal{N}(\mu_B, \sigma_B^2)$, the difference $A - B$ is Gaussian with

$$A - B \;\sim\; \mathcal{N}\big(\mu_A - \mu_B,\; \sigma_A^2 + \sigma_B^2\big).$$

Hence

$$\Pr[A > B] = \Pr[A - B > 0] = \Phi\left(\frac{\mu_A - \mu_B}{\sqrt{\sigma_A^2 + \sigma_B^2}}\right),$$

where Φ is the standard–normal CDF.

Fix $p \in (\frac{1}{2}, 1)$ and let $c := \Phi^{-1}(p)$, the unique value with $\Phi(c) = p$. Then:

$$\Pr[A > B] > p \quad\iff\quad \Phi\left(\frac{\mu_A - \mu_B}{\sqrt{\sigma_A^2 + \sigma_B^2}}\right) > p \quad\iff\quad \frac{\mu_A - \mu_B}{\sqrt{\sigma_A^2 + \sigma_B^2}} > c.$$

As the denominator $\sqrt{\sigma_A^2 + \sigma_B^2}$ is nonnegative, this is equivalent to

$$\Pr[A > B] > p \iff \mu_A - \mu_B > c\sqrt{\sigma_A^2 + \sigma_B^2}. \quad (3)$$

Now suppose $X \succ_p Y$ and $Y \succ_p Z$. Applying (3) gives

$$\mu_X - \mu_Y > c\sqrt{\sigma_X^2 + \sigma_Y^2} \qquad \text{and} \qquad \mu_Y - \mu_Z > c\sqrt{\sigma_Y^2 + \sigma_Z^2}.$$

Adding yields

$$\mu_X - \mu_Z > c\left(\sqrt{\sigma_X^2 + \sigma_Y^2} + \sqrt{\sigma_Y^2 + \sigma_Z^2}\right). \quad (4)$$

It remains to compare the RHS of (4) to $c\sqrt{\sigma_X^2 + \sigma_Z^2}$. Using $\sqrt{u} + \sqrt{v} \geq \sqrt{u+v}$ for all $u, v \geq 0$ and setting $u = \sigma_X^2 + \sigma_Y^2$ and $v = \sigma_Y^2 + \sigma_Z^2$, we get

$$\sqrt{\sigma_X^2 + \sigma_Y^2} + \sqrt{\sigma_Y^2 + \sigma_Z^2} \geq \sqrt{\sigma_X^2 + 2\sigma_Y^2 + \sigma_Z^2} \geq \sqrt{\sigma_X^2 + \sigma_Z^2}.$$

Combining with (4) implies

$$\mu_X - \mu_Z > c\sqrt{\sigma_X^2 + \sigma_Z^2}.$$

Applying (3) with $(A, B) = (X, Z)$ yields $\Pr[X > Z] > p$, i.e., $X \succ_p Z$. □